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A GENERALISED CENTRE OF A MONOID

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Abstract: **In this paper, the concept of monoid was examined on a generalised setting(multiset). Denoting the generalised setting of a monoid by a multi monoid, we introduced the concept of a multi-centre of a multi monoid and study the action of a centre of a multi monoid over the mset operations on the class of finite multi monoid. Further studies revealed that even though in general, the centre of a multi monoid need not be a multi monoid, however under the class of finite commutative multi monoids, the centre of a commutative multi monoid is a multi monoid and the centre of the commutative multi monoid is also a commutative multi monoid among other results.**

Keywords: **Monoid, Multiset, Multimonoid, Centre, Commutativity.**

1. INTRODUCTION.

George Ferdinand Ludwig Philip Cantor (1845-1918), a German mathematician is refered to as the father of set theory. In his cardinal key axiom, he stated that an element must belong to a set only once. However, as research grows, his theories could not address so many fundamental issues, such as the hydrogen molecules in water, DNA strand among other reasons. Thus the emergence of multiset (mset for short) which is a collection of objects with repetitions allowed. For the various applications of msets the reader is referred to article [1], [4,], [7], [9], and [11]. It is observed from the survey of available literature on msets and applications that the idea of mset was hinted by R. Dedikind in 1888. The mset theory which is termed as generalization of set theory was introduced by Cerf et al.[2] other literatures are [1,7,9,14]. The term mset, as noted by Knuth [4] was first suggested by N.G de Bruijn in a private communication to him. Further study was carried out by Yager [14], Blizard [1]. Other researchers ([5], [7], [8]) gave a new dimension to the mset theory.

Several authors have studied the structures of the classical sets under the generalised settings, such as: mset topological space [10], the concepts of relations, function, composition, and equivalence in msets context. [3], Tella and Daniel have considered sets of mappings between msets and studied about group and symmetric groups under mset perspective. ([12], [13]) Nazmul et al. improved on Tella and Daniel's work and added two axioms [6] In this paper we present the study of monoid in mset context while we lay more emphasis on the centre of the multi monoid. From the literatures, there may be some variations in the definition of monoid depending on the point of view of the different authors. However, in this paper we consider definitions in [15] and [16].

In addition to this section, we present some preliminary definitions in section two to make the paper self-contained and some fundamental results are presented in section three while the entire paper is summarized in section four.

2. PRELIMINARIES

2.1 Definitions and notations

Definition 2.1.1[15, 16]: Let S be a set and $\mu: S \times S \to S$ a binary operation that maps each ordered pair (x, y) of S to an element $\mu(x, y)$ of S. The pair (S, μ)(or just S, if there is no fear of confusion) is called a **groupoid**. The mapping μ is called the product of (S, μ). We shall mostly write simply xy Instead of $\mu(x, y)$. If we want to emphasize the place of the operation then we often write $x.y$. The element $xy(= \mu(x, y))$ is the product of x and y in S.

Definition 2.1.2[15, 16]: A groupoid S is a Semigroup if the operation μ is associative; for all $x\mu(y\mu z) = (x\mu y)\mu z, \forall\, x, y$ and $z \in S$. Thus a semigroup is a pair (S, μ) where S is a non empty set and μ is its binary operation on μ which satisfied two axioms:

(i) The closure property

(ii) The associativity property.

Definition 2.1.3[15,16](Monoid): A semi-group (S, μ) is a **monoid** if μ has an identity.

Definition 2.1.4[1]. An mset A over the set X can be defined as a function $C_A: X \to \mathbb{N} \cup \{0\}$, where $\mathbb{N} = \{0,1,2, \dots\}$ where the value $C_A(x)$ denote the number of times or multiplicity or count function of $x\ in\, A$. For example, Let $A = [x, x, x, y, y, y, z, z]$, then $C_A(x) = 3, C_A(y) = 3, C_A(z) = 2$. $[C_A(x) = 0 \Leftrightarrow x \notin A]$. If $C_M(x) = 0$ for all $x \in X$., then M is called an empty set. We denote the empty mset by $\emptyset$. Then $C_\emptyset(x) = 0, \forall\, x \in X$. ($C_A(x) > 0 \Leftrightarrow x \in A$)..If $C_A(x) = n$ then the membership of x in A can be denoted by $x \in^n A$, meaning x belong to A exactly n times.

Presentation of mset on paper work became a challenged as every researcher has his thought in that aspect. However the use of square brackets was adopted in ([1], [9],[11]) to represent an mset and ever since then it has become a standard. For example if the multiplicity of the elements x, y and z in an mset M are 2,3 and 2 respectively, then the mset M can be represented as $M = [x, x, y, y, y, z, z,]$, others put it like $[x, y, z]_{2,3,2}$ or $[x^2, y^3, z^2]$ or $[x2, y3, z2]$ or $[2/x, 3/y, 2/z]$ depending on one's taste or expediencies. But for conveniences sake, curly bracket can be used instead of the square bracket.

Definition 2.1.5[1]: The cardinality of a mset M denoted $|M|$ or $card(M)$ is the sum of all the multiplicities of its elements given by the expression $|M| = \sum_{x\in X} c_A(x)$

Note: An mset M is said to be finite if $|M| < \infty$.

We denote the class of all finite msets A over the set X by $M(X)$.

Definition 2.1.6[2]: Let M be an mset drawn from a set X. The support set of M denoted by M^* is a subset of X given by $M^* = \{x \epsilon X: C_M(x) > 0\}$. M^* is also called root set.

Definition 2.1.7[1](Equal msets): Two msets $A, B \in M(X)$ are said to be equal, denoted $A = B$ if and only if for any objects $x \in X, C_A(x) = C_B(x)$. This is to say that $A = B$ if the multiplicity of every element in A is equal to its multiplicity in B and conversely.

Definition 2.1.8[1] (Submultiset): Let $A, B \in M(X)$. A is a submultiset (submset for short) of B, denoted by $A \subseteq B\ or\ B \supseteq A$, if $C_A(x) \le C_B(x)\ for\ all\ x \in X$. Also if $A \subseteq B\ and\ A \ne B$, then A is called proper submset of B denoted by $A \subset B$. In other words $A \subset B$ if $A \subseteq B$ and there exist at least an $x \in X$ such that $C_A(x) < C_B(x)$. We assert that an mset B is called the parent mset in relation to the mset A.

Note that: For any two msets $A, B \in M(X)$, $A = B$ if and only if $A \subseteq B$ and. $B \subseteq A$.

Definition. 2.1.9[1]: (Regular or Constant mset): An mset$A$ over the set X is called regular or constant if all its elements are of the same multiplicities, i.e for any $x, y \in A$,

$$x \ne y \Rightarrow C_A(x) = C_A(y).$$

Definition 2.1.10: [9] (∧ and ∨ notations): The notations ∧ and ∨ denote the minimum and maximum operator respectively, for instance;

$$C_A(x) \wedge C_A(y) = min\{C_A(x), C_A(y)\}\ and\ C_A(x) \vee C_A(y) = max\{C_A(x), C_A(y)\}.$$

2.2 Multiset operations.

Definition 2.2.1[9] (msets union): Let $A, B \in M(X)$. The union of $A\ and\ B$ denoted $A \cup B$ is the mset defined by $C_{A\cup B}(x) = \{C_A(x) \vee C_B(x)\}\ \forall\, x \in X$,

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Definition 2.2.2[9] (msets intersection) Let$A, B \in M(X)$.The intersection of two mset A and B denoted by $A \cap B$, is the mset for which

$C_{A\cap B}(x) = \{C_A(x) \wedge C_B(x)\}\ \forall\ x \in X.$

Definition 2.2.3[9] (mset addition): Let $A, B \in M(X)$. The direct sum or arithmetic addition of A and B denoted by $A + B$ or $A \uplus B$ is the mset defined by

$C_{A+B}(x) = C_A(x) + C_B(x) \forall\ x \in X.$

Note that $|A \uplus B| = |A \cup B| + |A \cap B|$.

Definition 2.2.4[9] (mset difference): Let $A, B \in M(X)$, then the difference of B from A, denoted by $A - B$ is the mset such that $C_{A-B}(x) = \big(C_A(x) - C_B(x)\big) \vee 0, \forall\ x \in X$. If $B \subseteq A$, then

$C_{A-B}(x) = C_A(x) - C_B(x) \forall\ x \in X.$

It is sometimes called the arithmetic difference of B from A. If $B \nsubseteq A$ this definition still holds. It follows that the deletion of an element x from an mset A give rise to a new mset $A' = A - x$ such that $C_{A'}(x) = (C_A(x) - 1) \vee 0\ \ \forall\ x \in X$.

Definition 2.2.5[8] (mset symmetric difference): Let X be a set and $A, B \in M(X)$Then the symmetric difference, denoted $A \Delta B$, is defined by $C_{A\Delta B}(x) = |C_A(x) - C_B(x)|\ \forall\ x \in X$.

Note that $A \Delta B = (A - B) \cup (B - A)$.

Definition 2.2.6[8] (mset complement): Let $G = \{A_1, A_2, \ldots, A_n\}$ be a family of finite msets generated from the set X. Then, the maximum mset Z is defined by $C_Z(x) = max_{A\in G} C_A(x)$ for all $x \in X$. The Complement of an mset A, denoted by $\bar{A}$, is defined:

$\bar{A} = Z - A \text{ such that } C_{\bar{A}}(x) = C_Z(x) - C_A(x),\ \forall\ x \in X.$

Note that $A_i \subseteq Z \text{ for all } i$.

Definition 2.2.7[8] (Multiplication by Scalar): Let $A \in M(X)$, then the scalar multiplication denoted by $b.A$ is defined by $C_{b.A}(x) = b.C_A(x), \forall\ x \in X,$ where $b \in \{1,2,3, \ldots\}$.

Definition 2.2.8[8] (Arithmetic Multiplication): Let $A, B \in M(X)$, then the Arithmetic Multiplication denoted by $A.B$ is defined by $C_{A.B}(x) = C_A(x).C_B(x)\ \ \forall\ x \in X$.

Definition 2.2.9[7] (Raising to an Arithmetic Power): Let $A \in M(X)$, then A raised to power n denoted by A^n **is** defined:

$C_{A^n}(x) = \big(C_A(x)\big)^n\ \forall\ x \in X,\ \ n \in \{0,1,2,3, \ldots\}.$

Definition 2.3.10[19]: Let X be a groupoid, and $A \in M(X)$. A is said to be a multi-groupoid (mgroupoid for short) if the following condition is satisfied.

$C_A(xy) \geq C_A(x) \wedge C_A(y), \forall\ x, y\ \in X.$

We denote the class of all finite mgroupoids over X by $MGP(X)$.

Definition 2.3.11[20]: Let $A \in MGP(X)$, then A is said to be a semi –multigroup (semi-mgroup for short) if X is a semi-group.

We denote the class of all finite semi-mgroups over X by *SMG(X)*.

Clearly $SMG(X) \subset MGP(X)$.

Definition 2.3.12[20]: Let $A \in SMG(X)$ and let B be a submset of A. Then B can be said to be a sub semi-mgroup of A, if $B \in SMG(X)$.

Definition 2.3.13[21] Let $A \in SMG(X)$. Then A is said to be a multimonoid (mmonoid for short). If

(i) X is a monoid and

(ii) $C_A(e) \geq C_A(x) \forall\, x \in X$.

Where e is the identity element in X.

Let the class of all finite mmonoids over the monoid X be denoted as $MM(X)$ such that $A \neq \emptyset$..

Definition 2.3.14: Let $A \in MM(X)$ and let B be a submset of A. Then B is said to be a sub mmonoid of A, if $B \in MM(X)$.

Definition 2.3.15[21]; Composition of mmonoid: Let $A, B \in MM(X)$, then we call

$A \circ B$ the composition of A and B defined

$C_{A \circ B}(x) = \vee\{C_A(y) \wedge C_B(z): y, z \in X \ni yz = x\}$.

Theorem 2.3.16[21]: $MM(X) \subset SMG(X)$.

Theorem 2.3.17[21] Let $A \in MM(X)$. Then A^* is a sub monoid of X.

Theorem 2.3.18[21]: Let $A \in MM(X)$, then $A^* \in MM(X)$.

Theorem 2.3.19[21]: Let $A, B \in MM(X)$, Then

(i) $A \cap B \in MM(X)$.

(ii) $k.A \in MM(X)$, $k \in \{1,2 \ldots.\}$

(iii) $A.B \in MM(X)$

(iv) $A^n \in MM(X), n \in \{0,1,2,\ldots\}$

(v) $AoB \in MM(X)$

Definition 2.3.20[21]: Let $A \in MM(X)$. Then A is said to be cancellable if it is cancellable semi-mgroup.

We denote the class of finite cancellable mmonoid as $\mathbb{C}MM(X)$.

That is $\mathbb{C}MM(X) = \{A \in MM(X) \mid A$ is cancellable$\}$.

Definition 2.3.21[21]: Let $A \in MM(X)$, then A is said to be a commutative mmonoid if it is a commutative semi-mroup.

Commutative mmonoid can also be called Abelian mmonoid.

We denote the class of finite commutative mmonoid as $CMM(X)$.

That is $CMM(X) = \{A \in MM(X) | A$ is commutative$\}$

Theorem 2.3.22[21]: Let $A \in MM(X)$. If A is a commutative mmonoid, then A^* is a commutative sub mmonoid.

Theorem 2.3.23[21]: Let $A, B \in MM(X)$ such that A and B are commutative. Then the following expressions are commutative.

(i) $A \cap B$

(ii) $A \cup B$

(iii) $A + B$

(iv) $A - B$

(v) $A \Delta B$

(vi) $A \cdot B$

(vii) $kA, k \in \{1,2,3,\ldots\}$

(viii) A^n, $n \in \{0,1,2,\ldots\}$

(ix) AoB

Theorem 2.3.24[21]: Let $A, B \in CMM(X)$. Then

(i) $A \cap B \in CMM(X)$.

(ii) $k.A \in CMM(X)\ k \in \{1,2 \dots.\}$

(iii) $A.B \in CMM(X)$

(iv) $A^n \in CMM, n \in \{0,1,2,\dots\}$

(v) $AoB \in CMM(X)$

Theorem 2.3.25[21]: $\mathbb{C}MM(X) = CMM(X)$.

3. CENTRE OF MMONOIDS

Definition 3.1.1: Let X be a monoid and $A \in MM(X)$. We defined the center of A, denoted $Z(A)$ as

$Z(A) = \{x \in^n A \mid C_A(xy) = C_A(yx)\ \} \forall\, y \in X.$

Example 3.1.2: Let $X = \{1, -1, i, -i\}$ a monoid , where 1 is the identity element, under the multiplicative operation, and let $A = [1, -1, i, -i]_{3,2,2,1}$. Here,

$$C_A(1.-1) = C_A(-1) = 2 = C_A(-1.1) = C_A(-1) = 2$$
$$C_A(1.i) = C_A(i) = 2 = C_A(i.1) = C_A(i) = 2$$
$$C_A(1.-i) = C_A(-i) = 1 = C_A(-i.1) = C_A(-i) = 1$$
$$C_A(-1.i) = C_A(-i) = 1 = C_A(i.-1) = C_A(-i) = 1$$
$$C_A(-1.-i) = C_A(i) = 2 = C_A(-i.-1) = C_A(i) = 2$$
$$C_A(i.-i) = C_A(1) = 3 = C_A(-i.i) = C_A(1) = 3$$
$$C_A(1.1) = C_A(1) = 3 = C_A(1.1) = C_A(1) = 3$$
$$C_A(-1.-1) = C_A(1) = 3 = C_A(-1.-1) = C_A(1) = 3$$
$$C_A(i.i) = C_A(-1) = 1 = C_A(i.i) = C_A(-1) = 1$$
$$C_A(1.-1) = C_A(-1) = 1 = C_A(-1.1) = C_A(-1) = 1$$
$$C_A(-i.-i) = C_A(-1) = 1 = C_A(-i.-i) = C_A(-1) = 1$$

In particular, $Z(A) = [1, -1, i, -i]_{3,2,2,1}$

Thus $Z(A)$ is the center of A.

Proposition 3.1.3: Let $A \in MM(X)$ such that A is commutative. Then $Z(A) = A$.

Proof: Since A is commutative, we have

$C_A(xy) = C_A(yx)\ \ \forall\, x, y \in X$

Thus for any $x \in X$, we have $x \in Z(A)$

In particular, $C_{Z(A)}(x) = C_A(x)\ \ \forall\, x \in X$(by definition)

Thus $Z(A) = A$.

Proposition 3.1.4: Let $A \in MM(X)$, such that A is commutative. Then $Z(A) \in MM(X))$.

Proof; Clearly X is a semi-group (1)

Then we show that $C_{Z(A)}(xy) \geq C_{Z(A)}(x) \wedge C_{Z(A)}(y)$ and $C_{Z(A)}(e) \geq C_{Z(A)}(x)\ \forall\, x \in X$.

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Now $A \in MM(X)$ implies $C_A(xy) \geq C_A(x) \wedge C_A(y)$ (2)

But for any $x, y \in X$, we have $xy \in X$

$C_A(xy) = C_A(yx)$ $\forall\, x, y \in X$(since A is commutative), and

$C_{Z(A)}(xy) = C_A(xy) \geq C_A(x) \wedge C_A(y)$ (3)(from 2)

But $x, y, z \in Z(A)$

$C_{Z(A)}(xy) = C_A(xy) \geq C_{Z(A)}(x) \wedge C_{Z(A)}(y)$ (from 3)

Therefore, $C_{Z(A)}(xy) \geq C_{Z(A)}(x) \wedge C_{Z(A)}(y)\ \forall\, x \in X$ (4)

Since $C_A(xe) = C_A(ex)$ $\forall\, x \in X.$ Thus $e \in Z(A)$.

$C_{Z(A)}(e) = C_A(e) \geq C_A(x)$ $\forall\, x \in X$, by hypothesis

i.e $C_{Z(A)}(e) \geq C_{Z(A)}(x)$ $\forall\, x \in X$. Since from (5)

From (4) and (5) above, if A is commutative then $Z(A) \in MM(X)$.

Hence the result.

Proposition 3.1.5: Let $A \in MM(X)$ such that A is commutative. Then $Z(A)$ is commutative.

Proof: From definition 3.1.1 and proposition 3.1.3. The result follows. Then

$C_{Z(A)}(xy) = C_A(xy) = C_A(yx) = C_{Z(A)}(yx)$.

Thus $C_{Z(A)}(xy) = C_{Z(A)}(yx)$.

In particular, $Z(A)$ is commutative.

Proposition 3.1.6: Let $A \in CMM(X)$. If A is cancellable, then $Z(A)$ is cancellable.

Proof: Let $A \in CMM(X)$, then $Z(A)$ is commutative(Proposition 3.1.3)

Thus $Z(A)$ is cancellable(see [20]).

Proposition 3.1.7: Let $A \in MM(X)$. Then A is commutative, if and only if $Z(A)$ is cancellable.

Proof: Assuming A is commutative. then $Z(A)$ is commutative(proposition 3.1.5).

Thus $Z(A)$ is cancellable(see [20]).

Proposition 3.1.8: Let $A, B \in CMM(X)$, then

(i) $Z(A \cap B) \in CMM(X)$.

(ii) $k.Z(A) \in CMM(X)$, $k \in \{1,2 \dots.\}$

(iii) $Z(A.B) \in CMM(X)$

(iv) $Z(A)^n \in CMM(X), n \in \{0,1,2, \dots\}$

(v) $Z(AoB) \in CMM(X)$

Proof:

(i)Since $A, B \in CMM(X)$, then $A \cap B \in CMM(X)$(proposition 2.3.24). In particular

$Z(A \cap B) \in CMM(X)$.By proposition 3.1.3

(ii)Since $A \in CMM(X)$, then $k.A \in CMM(X)$(proposition 2.3.24). In particular $k.Z(A) \in CMM(X)$.By proposition 3.1.3

(iii)Since $A, B \in CMM(X)$. then $A.B \in CMM(X)$(proposition 2.3.24). In particular

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$Z(A.B) \in CMM(X)$.By proposition 3.1.3

(iv)Since $A, B \in CMM(X)$, then $A^n \in CMM(X)$(proposition 2.3.24). In particular $Z(A)^n \in CMM(X)$.By proposition 3.1.3

(v)Since $A, B \in CMM(X)$, then $AoB \in CMM(X)$(proposition 2.3.24). In particular

$Z(AoB) \in CMM(X)$.By proposition 3.1.3

Proposition 3.1.9: Let $A, B \in CMM(X)$. Then

(i) $Z(A \cap B) = Z(A) \cap Z(B)$

(ii) $Z(A \cup B) = Z(A) \cup Z(B)$

(iii) $Z(A + B) = Z(A) + Z(B)$

(iv) $Z(A - B) = Z(A) - Z(B)$

(v) $Z(A\Delta B) = Z(A)\Delta Z(B)$

(vi) $Z(A \cdot B) = Z(A) \cdot Z(B)$

(vii) $Z(k.A) = k\, Z(A), k \in \{1,2,3, \dots\}$

(viii) $Z(A^n) = (Z(A))^n,\ n \in \{0,1,2, \dots\}$

(ix) $Z(Ao\, B) = Z(A)o\, \mathrm{Z}(B)$

Proof:

(i) $Z(A \cap B) = A \cap B$ (since $A \cap B$ is commutative) (see [21])

and $A \cap B = Z(A) \cap Z(B)$(proposition 3.1.3)

In particular, $Z(A \cap B) = Z(A) \cap Z(B)$.

(ii) $Z(A \cup B) = A \cup B$ (since $A \cup B$ is commutative) (see [21])

and $A \cup B = Z(A) \cup Z(B)$(proposition 3.1.3)

Thus $Z(A \cup B) = Z(A) \cup Z(B)$.

(iii) $Z(A + B) = A + B$ (since $A + B$ is commutative) (see [21])

and $A + B = Z(A) + Z(B)$(proposition 3.1.3)

Thus $Z(A + B) = Z(A) + Z(B)$.

(iv) $Z(A - B) = A - B$ (since $A - B$ is commutative) (see [21])

and $A - B = Z(A) - Z(B)$(proposition 3.1.3)

Thus $Z(A - B) = Z(A) - Z(B)$.

(v) $Z(A\Delta\, B) = A\Delta\, B$ (since $A\Delta\, B$ is commutative) (see [21])

and $A\Delta\, B = Z(A)\Delta Z(B)$(proposition 3.1.3)

Thus $Z(A\Delta B) = Z(A)\Delta Z(B)$

(vi) $Z(A.B) = A.B$ (since $A.B$ is commutative) (see [21])

and $A.B = Z(A).Z(B)$(proposition 3.1.3).

Thus $Z(A.B) = Z(A).Z(B)$

(vii) $Z(k.A) = k.A$(since $k.A$ is commutative) (see [21]) and $k.A = k.Z(A)$(proposition 3.1.3)

Thus $Z(k.A) = \text{k}.Z(A)$.

(viii) $Z(A^n) = A^n$ (since A^n is commutative) (see [21])

and $A^n = (Z(A))^n$ (proposition 3.1.3)

Thus $Z(A^n) = (Z(A))^n$.

(ix) $Z(Ao\ B) = Ao\ B$(since $Ao\ B$ is commutative) (see [21])

and $Ao\ B = Z(A)oZ(B)$(proposition 3.1.3).

Thus $Z(AoB) = Z(A)oZ(B)$

Proposition 3.1.10: Let $A, B \in CMM(X)$. Then the following expressions are commutative:

(i) $Z(A \cap B)$

(ii) $Z(A \cup B)$

(iii) $Z(A + B)$

(iv) $Z(A - B)$

(v) $Z(A\Delta B)$

(vi) $Z(A \cdot B)$

(vii) $k\ Z(A), k \in \{1,2,3, \dots\}$

(viii) $Z(A^n),\ n \in \{0,1,2, \dots\}$

(ix) $Z(Ao\ B)$

Proof: Since $A \cap B$, $A \cup B$, $A + B$. $A - B$, $A\Delta B$, $A.B, k.A$, A^n and $Ao\ B$ are all commutative expressions (From Theorem 2.3.23) It is clear that the expressions (i) to (ix) are all commutative expressions(proposition 3.1.3).

Proposition 3.1.11: Let $A, B \in CMM(X)$. Then the following expressions are cancellable:

(i) $Z(A \cap B)$

(ii) $Z(A \cup B)$

(iii) $Z(A + B)$

(iv) $Z(A - B)$

(v) $Z(A\Delta B)$

(vi) $Z(A \cdot B)$

(vii) $k\ Z(A), k \in \{1,2,3, \dots\}$

(viii) $Z(A^n),\ n \in \{0,1,2, \dots\}$

(ix) $Z(Ao\ B)$

Proof: Since $A \cap B, A \cup B$, $A + B$. $A - B$, $A\Delta B$, $A.B, k.A$, A^n and $Ao\ B$ are all cancellable expressions (See [21]) It is clear that the expressions (i) to (ix) are all cancellable expressions(Proposition 3.1.3).

Proposition 3.1.12: Let $A, B \in \mathbb{C}MM(X)$. Then

(i) $Z(A \cap B) \in \mathbb{C}MM(X)$.

(ii) $k.Z(A) \in \mathbb{C}MM(X)$, $k \in \{1,2 \dots.\}$

(iii) $Z(A.B) \in \mathbb{C}MM(X)$

(iv) $Z(A)^n \in \mathbb{C}MM(X), n \in \{0,1,2, \dots\}$

(v) $Z(AoB) \in \mathbb{C}MM(X)$

Proof: From theorem 2.3.25, proposition 3.1.7 and proposition 3.1.8 The result is clear.

4. CONCLUSION

In this paper, the concept of monoid was examined on a generalised setting (multiset). Denoting the generalised setting of a monoid by a multi monoid, we introduced the concept of a centre of a multi monoid and study the action of a centre of a multi monoid over the mset operations on the class of finite multi monoid. We have shown that the centre of intersection, arithmetic multiplication, composition, raising to an arithmetic power and multiplication by scalar are closed over the class of finite commutative multi monoids. Further studies revealed that even though in general, the centre of an multi monoid is not a multi monoid, however, under the class of finite commutative multi monoid, the centre of a commutative multi monoid is the multi monoid, among other results.

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